Comparison Summary: Spreadsheets vs. Our GPS Calculator

I've analyzed both Excel spreadsheets and compared them to our HTML GPS Positional Tolerance Calculator. Here's what I found:

📊 Core Calculations Comparison

Calculation	Spreadsheets	Our Calculator	Match?
Horizontal Error	`sqrt((X-Xref)² + (Y-Yref)²)`	`sqrt((X-Xmean)² + (Y-Ymean)²)`	✅ Same formula
Vertical Error	`abs(Z - Zref)`	`abs(Z - Zmean)`	✅ Same formula
Reference Point	Manual entry or mean	Always uses mean	⚠️ More consistent
Units	Converts meters → survey feet	Direct feet input	✅ Compatible

📈 Statistical Analysis Comparison

Feature	Spreadsheets	Our Calculator	Advantage
95% Confidence	Simple AVERAGE	True 95th percentile (CEP 95%)	✅ Ours is statistically correct
RMS Calculation	Not present	`sqrt(Σ(errors²)/n)`	✅ Industry standard
CEP 50%	Not calculated	50th percentile	✅ Additional metric
Outlier Detection	None	Grubbs test	✅ Statistical rigor

🔧 Functionality Comparison

Feature	Spreadsheets	Our Calculator
Automatic Grouping	❌ Manual only	✅ Name/Distance/Hybrid
NC Standards Check	❌ Not present	✅ Full compliance checking
Large Datasets	⚠️ Limited (Excel rows)	✅ Handles 20,000+ points
Visualization	❌ None	✅ Scatter plot & histogram
Export Options	⚠️ Excel only	✅ CSV with full statistics

🎯 Key Findings

1. **Core Math is Identical**: Both use the same fundamental error calculation formulas
2. **Statistical Rigor**: Our calculator provides true percentile-based statistics vs. simple averaging
3. **The "95% Confidence" Misconception**: The spreadsheets label their average as "95% confidence" but don't calculate actual 95th percentiles
4. **Enhanced Features**: Our calculator adds grouping, NC standards compliance, and visualizations

✅ Validation Result

The core mathematical calculations in our GPS Positional Tolerance Calculator are **correct and match** the spreadsheet formulas. However, our implementation is **more statistically rigorous** by using:

- True percentile calculations (CEP 50% and 95%)
- RMS instead of simple averaging
- Proper statistical outlier detection
- Automated analysis features

The spreadsheets appear to be older tools designed for manual OPUS data processing, while our calculator provides a modern, automated approach with proper statistical methods and compliance checking.

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GPS Tolerance Calculator - Mathematical Reference Documentation

Overview

This document describes all mathematical calculations and algorithms used in the GPS Positional Tolerance Calculator with North Carolina surveying standards compliance.

---

1. Core Statistical Calculations

1.1 Mean Position Calculation

For a set of points, the mean position is calculated as:

- **Mean X** = Σ(Xi) / n
- **Mean Y** = Σ(Yi) / n
- **Mean Z** = Σ(Zi) / n

Where n = number of points

1.2 Error Calculations for Each Point

For each point relative to the mean position:

- **ΔX** = Xi - Mean X
- **ΔY** = Yi - Mean Y
- **ΔZ** = Zi - Mean Z
- **Horizontal Error** = $\sqrt{(\Delta X^2 + \Delta Y^2)}$
- **Vertical Error** = $|\Delta Z|$
- **Total Error** = $\sqrt{(\Delta X^2 + \Delta Y^2 + \Delta Z^2)}$

1.3 Root Mean Square (RMS) Calculations

- **RMS Horizontal** = $\sqrt{(\Sigma(\text{Horizontal Error}^2) / n)}$
- **RMS Vertical** = $\sqrt{(\Sigma(\text{Vertical Error}^2) / n)}$

1.4 Circular/Vertical Error Probable (CEP/VEP)

1. Sort all horizontal errors from smallest to largest
2. **CEP 50%** = error value at 50th percentile position
3. **CEP 95%** = error value at 95th percentile position

4. Same process for vertical errors to get VEP 50% and VEP 95%

Percentile Calculation:

- For n points, 95th percentile position = floor(n × 0.95)
- If n = 100, the 95th percentile is the 95th smallest error value

---

2. North Carolina Standards Classification

2.1 Horizontal Classification (21 NCAC 56 .1603)

Boundary Survey Standards:

- Class AA: Base Error ≤ 0.05 ft + 30 ppm
- Class A: Base Error ≤ 0.10 ft + 50 ppm
- Class B: Base Error ≤ 0.12 ft + 90 ppm
- Class C: Base Error ≤ 0.15 ft + 150 ppm

PPM Calculation:

- Allowable Error = Base Error + (ppm × distance / 1,000,000)
- Distance is typically 0 for single point analysis

Topographic Standards (used as fallback):

- Class AA: ≤ 0.10 ft
- Class A: ≤ 1.64 ft
- Class B: ≤ 3.28 ft
- Class C: ≤ 6.56 ft
- Class D: ≤ 16.40 ft

2.2 Vertical Classification (21 NCAC 56 .1606)

Level Run Standards:

- Class A: Error ≤ 0.10 × √(miles)
- Class B: Error ≤ 0.20 × √(miles)
- Class C: Error ≤ 0.30 × √(miles)

Distance Calculation:

- Distance from base (ft) = $\sqrt{((X - BaseX)^2 + (Y - BaseY)^2)}$
- Miles = Distance / 5280

- Allowable Error = Factor × √(miles)

Note: Vertical classification requires a reference base point

---

3. Project Classification Determination

3.1 Overall Horizontal Classification

- Use **CEP 95%** value (not average or worst error)
- Compare CEP 95% against NC standards thresholds
- Project meets the highest class where CEP 95% ≤ allowable error

3.2 Overall Vertical Classification

- Use **VEP 95%** value
- Calculate average distance from base for all points
- Compare VEP 95% against allowable error for that distance

---

4. Grouping Algorithms

4.1 Name-Based Grouping

- Points with names ending in `.N` (where N is a number) are grouped
- Example: `cntrl1.1, cntrl1.2, cntrl1.3` → group "cntrl1"
- Regex pattern: `/\.\d+$/`

4.2 Distance-Based Grouping

1. Start with first unassigned point as group seed
2. Find all points within radius R of any point in current group
3. Repeat until no new points found within radius
4. Move to next unassigned point and create new group
5. Default radius = 0.30 ft

4.3 Hybrid Grouping

1. First group by name pattern
2. Then check if points in same name group are within distance radius
3. If not, split into subgroups (e.g., `cntrl1_1`, `cntrl1_2`)

---

5. Grouped Data Analysis

5.1 Within-Group Statistics (Precision)

For each group:

1. Calculate group mean position ($\bar{X}g$, $\bar{Y}g$, $\bar{Z}g$)
2. Calculate errors for each point relative to group mean
3. Calculate group RMS, CEP, VEP statistics
4. Classify group based on its CEP 95%

5.2 Between-Group Statistics (Accuracy)

For multiple groups:

1. Calculate mean of group means (overall center)
2. Calculate each group's error from overall center
3. These represent control point distribution across project

5.3 Overall Project Statistics for Grouped Data

1. Pool ALL individual point errors from ALL groups
2. Sort combined errors and find overall CEP 95%
3. This determines project classification (not group averages)

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6. Statistical Tests

6.1 Grubbs Test for Outliers

Purpose: Detect statistical outliers

Calculation:

1. Calculate mean (μ) and standard deviation (σ) of errors
2. Find most extreme value: $\max|Xi - \mu|$
3. Grubbs statistic $G = \max|Xi - \mu| / \sigma$
4. Critical value $Gcrit = ((n-1)/\sqrt{n}) \times \sqrt{(t^2/(n-2+t^2))}$
5. If $G > Gcrit$, outlier detected at significance level α

t-critical values (simplified):

- $\alpha = 0.05$: varies by degrees of freedom (n-2)
- Default to $t = 1.96$ for large samples

6.2 Chi-Square Normality Test

Purpose: Test if errors follow normal distribution

Process:

1. Create histogram bins (typically 10-20)
2. Count observed frequencies in each bin
3. Calculate expected frequencies for normal distribution
4. Chi-square statistic: $\chi^2 = \Sigma((\text{Observed} - \text{Expected})^2/\text{Expected})$
5. Degrees of freedom = bins - 3 (for estimated μ and σ)
6. Compare to critical value

Note: GPS errors typically follow Rayleigh distribution, not normal

---

7. Error Function (erf)

Used for normal distribution calculations:

$$\text{erf}(x) = (2/\sqrt{\pi}) \int_0^x e^{(-t^2)}\, dt$$

Approximation used:

$$\text{erf}(x) = \text{sign}(x) \times (1 - (a_1t + a_2t^2 + a_3t^3 + a_4t^4 + a_5t^5) \times e^{(-x^2)})$$
$$\text{where } t = 1/(1 + p|x|)$$

Constants:

- $a_1 = 0.254829592$
- $a_2 = -0.284496736$
- $a_3 = 1.421413741$
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8. Visualization Calculations

8.1 Scatter Plot

- Scale factors: (canvas width - padding) / (max - min)
- Point colors based on error/CEP95 ratio:
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- 68% confidence: ~1 standard deviation
- 95% confidence: ~2 standard deviations (used for classification)
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9.3 Unit Conversions

- Feet to meters: multiply by 0.3048
- Feet to miles: divide by 5280

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10. Export Calculations

All calculated values are preserved in CSV export with appropriate precision:

- Coordinates: 3 decimal places
- Errors: 3 decimal places
- Distances: 1 decimal place
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Summary

The key mathematical principle is that **project classification is based on the 95th percentile error (CEP 95%)**, not averages or worst-case errors. This ensures that 95% of measurements meet the specified standard, allowing for realistic field conditions while maintaining quality standards as required by North Carolina surveying regulations.

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